Ch. 2. Origin of Quantum theory

1. Failure of Classical Mechanics:

Before 19^{th} century, most of phenomena could be explained on the basis of classical physics which is based on Newton's three laws: 1) law of inertia, ii) law of force and iii) law of action and reaction. The equation based on these laws are simple and can explain successfully the motion of objects which are either directly observable or can be made observable by simple instruments like microscope. In this way, the classical mechanics explained successfully the motion of bodies moving with non-relativistic speed i.e. $V \ll C$.

With discovery of electron, explorations of microscopic systems were started. The classical concepts cannot be directly applied to the motion of electrons in an atom which are not observable with help of instruments. Various difficulties arise in explaining the various phenomenon on basis of classical mechanics such as, i) stability of atom, ii) spectral distribution of heat radiation from black bodies, ii) photoelectric effect, iv) specific heat of solids at low temperatures vi) Compton effect.

The electrons revolving round the nucleus have centripetal acceleration. According to classical theory every accelerated charged particles radiates energy in the form of electromagnetic waves. Hence orbiting electrons radiates energy continuously. Due to this, there is continuous loss of energy of electrons; radii of their orbits should fall into nucleus. Thus atom cannot remain stable. Thus classical physics failed to explain stability of atom.

Max plank in 1900 introduce the new concepts that absorption or emission of electromagnetic radiation takes place as discrete quanta, each having energy $E=h\nu$ where ν is frequency of radiation and h is plank's constant. These concepts led to a new mechanics which is known as quantum mechanics.

2. Black Body Radiation:

A body which absorbs all the heat radiations incident on it is called as black body. When such a body is placed inside an isothermal enclosure, it will emit the full radiations of the enclosure after it is in equilibrium with the enclosure. These radiations are independent of the nature of the substance. Such heat radiations in a uniform temperature enclosure are known as black body radiation. Also the black body completely absorbs the heat radiations of all wavelengths. Thus the black body also emits completely the radiations of all wavelengths at that temperature.

In practice, a perfectly black body does not exist in nature but lamp black is considered as perfectly black body. The construction of perfectly black body is as shown in figure (a).

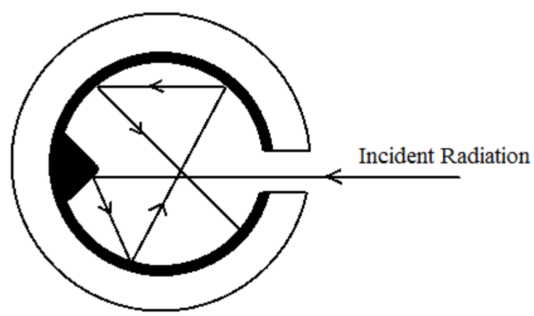

Fig. (a)

It consists of a double walled hollow sphere having a small aperature, through which heat radiation can enter. The space between the walls is evacuated and outer surface of the sphere is silvered. The inner surface of sphere is coated with lamp black and it has a conical projection diameterally opposite to the aperature. The projection ensures that a ray travelling along the axis of the aperature is not incident normally on the surface and is therefore not reflected back along the same path. The radiant heat entering the sphere suffers multiple reflections. During each reflections near about 98% of heta is absorbed. This body acts as a black body absorber. When this body is placed in a bath at a fixed temperature, the heat radiations are given out of aperature. A aperature acts as a black body radiator.

Distribution of energy in the spectrum of a black body:

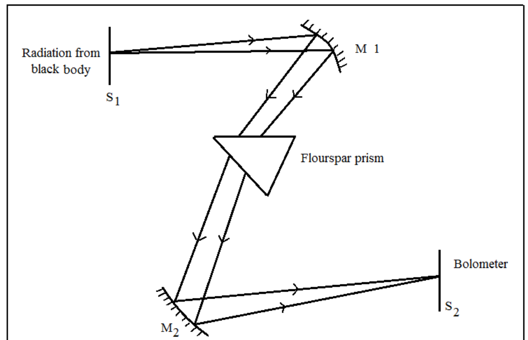

Fig. (b)

The distribution of energy in various parts of the spectrum was experimentally studied by Lummer and Pringsheim. The experimental arrangement is as shown in Fig. (b). The radiations from black body are allowed to be incident on concave mirror M_1. It is then allowed to fall on a prism of fluorspar to resolve it into a spectrum. The spectrum is brought to focus by another concave mirror M_2 on to a linear bolometer. The bolometer is connected to a galvanometer. The deflections in galvanometer corresponding to different wavelength λ are noted by rotating prism table. Then curves are plotted for intensity of radiation E_λ with wavelength λ. The experiment is done with black body at different temperatures. The curve obtained is as shown in Fig. (c)

From Fig. (c) following results are obtained:

i) At a given temperature, the energy is not uniformly distributed in the radiation spectrum of a hot body.

ii) At any given temperature, the intensity of radiation E_λ first increases with wavelength λ, reaching a maximum value corresponding to a particular wavelength λ_m and then decreases for longer wavelengths.

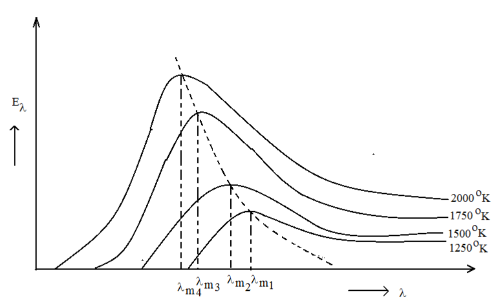

Fig. (c)

iii) The value of E_λ for any λ increases as temperature increases.

iv) The wavelength corresponding to maximum energy shifts to shorter wavelength side as temperature increases. This confirms Wien's displacement law $\lambda_m T = Constant$.

Wien's displacement law state that the wavelength of most strongly emitted radiation in the continuous spectrum from a full radiator is inversely proportional to absolute temperature of that body i.e. $\lambda_m T = b$ where b is Wien's constant and it is 2.898×10^{-3} mk.

v) The area under each curve represents the total energy emitted for the complete spectrum of a particular temperature. This area is found to be proportional to the fourth power of absolute temperature. This verifies Stefan's law.

3. Plank's quantum theory:

According to classical theory of radiation, energy changes of radiators take place continuously. The classical theory failed to explain experimentally distribution of energy in spectrum of a black body. Plank put forward the quantum theory to explain the experimentally observed spectrum of black body radiation. He suggested the quantum theory of radiations. His postulates are:

i) A black body radiation chamber is filled up not only with radiation but also with simple harmonic oscillator or resonators of molecular dimensions. They can vibrate with all possible frequencies.

ii) The oscillators or resonators cannot radiate or absorb energy continuously. But an oscillator of frequency ν can only radiate or absorb energy in form of quanta. Each quanta have energy $h\nu$ where h is plank's constant.

iii) The emission of radiation corresponds to decrease and absorption to an increase in the energy and amplitude of an oscillator.

Plank's Radiation Law:

Planck's law for the energy E_λ radiated per unit volume by a cavity of a blackbody in the wavelength interval λ to λ + Δλ can be written in terms of Planck's constant (*h*), the speed of light (*c*), the Boltzmann constant (*k*), and the absolute temperature (*T*) :

$$E_\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp\left({hc}/{\lambda kT}\right) - 1}$$

Derivation of Plank's Radiation Law:

Let N be total number of Plank's resonators and E their total energy.

Then, the average energy per oscillator is

$$\bar{\varepsilon} = \frac{E}{N}$$

Let N_0, N_1, N_2, -------- N_r ----- etc be number of resonators having energy, 0, ε, 2ε, ----- $r\varepsilon$-----etc respectively. Then, we have

$N = N_0 + N_1 + N_2 + \text{-----} + N_r + \text{-----}$

And

$E = 0 + \varepsilon N_1 + 2\varepsilon N_2 + \text{-------}\ r\,\varepsilon N_r + \text{-------}$

$\quad = \varepsilon\,(N_1 + 2N_2 + \text{-----} + r\,N_r + \text{-----})$

According to Maxwell's distribution formula, the No. of resonators having energy $r\varepsilon$ will be,

$$N = N_0 e^{-r\varepsilon/kT}$$

Where r = 0, 1, 2, 3-------

$$N = N_0 + N_0 e^{-\varepsilon/kT} + N_0 e^{-2\varepsilon/kT} + --- + N_0 e^{-r\varepsilon/kT} + ----$$

$$= N_0\left(1 + e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} + --- + e^{-r\varepsilon/kT} + ----\right)$$

Put $\varepsilon/kT = x$

$$\therefore N = N_0[1 + e^{-x} + e^{-2x} + ------ + e^{-rx} + ----]$$

$$\therefore N = \frac{N_0}{1 - e^{-x}} \quad -----------(1)$$

Total energy of Plank's resonators is,

$$E = 0 \times N_0 + \varepsilon N_0 e^{-x} + 2\varepsilon N_0 e^{-2x} + --- + r\varepsilon N_0 e^{-rx} + ---$$

$$E = \varepsilon N_0 e^{-x} + 2\varepsilon N_0 e^{-2x} + --- + r\varepsilon N_0 e^{-rx} + ---$$

$$Ee^{-x} = \varepsilon N_0 e^{-2x} + 2\varepsilon N_0 e^{-3x} + --- + r\varepsilon N_0 e^{-(r+1)x} + ---$$

Then,

$$E - Ee^{-x} = [\varepsilon N_0 e^{-x} + 2\varepsilon N_0 e^{-2x} + --- + r\varepsilon N_0 e^{-rx} + ---]$$

$$- \left[\varepsilon N_0 e^{-2x} + 2\varepsilon N_0 e^{-3x} + --- + r\varepsilon N_0 e^{-(r+1)x} + ---\right]$$

$$E(1 - e^{-x}) = \varepsilon N_0 e^{-x} + \varepsilon N_0 e^{-2x} + \varepsilon N_0 e^{-3x} \ ---$$

$$= \varepsilon N_0 (e^{-x} + e^{-2x} + e^{-3x} + ---- = \frac{\varepsilon N_0 e^{-x}}{1 - e^{-x}}$$

$$E = \frac{\varepsilon N_0 e^{-x}}{(1 - e^{-x})^2} \ ------------(2)$$

Average energy of a resonator is

$$\bar{\varepsilon} = \frac{E}{N} = \frac{\varepsilon N_0 e^{-x}}{(1 - e^{-x})^2} \times \frac{(1 - e^{-x})}{N_0}$$

$$= \frac{\varepsilon e^{-x}}{1 - e^{-x}} = \frac{\varepsilon}{\frac{1}{e^{-x}} - 1} = \frac{\varepsilon}{e^{x} - 1}$$

According to Plank's hypothesis, E = $h\nu$

we have $\nu = \frac{C}{\lambda} \quad \therefore \varepsilon = \frac{hc}{\lambda}$ $and\ x = \frac{\varepsilon}{kT} \qquad \therefore x = \frac{hc}{\lambda kT}$

$$\therefore \bar{\varepsilon} = \frac{hc/\lambda}{\left(e^{hc/\lambda kT} - 1\right)} \ -------- (3)$$

Number of oscillations per unit volume in wavelength range λ and λ+dλ is $8\pi\lambda^{-4}d\lambda$.

Hence energy density of radiation between wavelength λ and λ+dλ is

$E_\lambda . d\lambda$ = (average energy of a plank's oscillator) × (No. Of oscillations per unit volume)

$$E_\lambda d\lambda = \frac{hc/\lambda}{\left(e^{hc/\lambda kT} - 1\right)} \times 8\pi\lambda^{-4} d\lambda$$

$$E_\lambda d\lambda = \frac{8\pi hc\lambda^{-5} d\lambda}{\left(e^{hc/\lambda kT} - 1\right)} -------- (4)$$

Eq[n] (4) represents plank's radiation law in terms of wavelength.

$$\text{But } \nu = \frac{c}{\lambda} \quad \therefore \lambda = \frac{c}{\nu} \quad \text{and} \quad d\lambda = \frac{-c}{\nu^2} d\nu \quad |d\lambda| = \frac{c}{\nu^2} d\nu$$

$$E_\nu d\nu = \frac{8\pi hc \left(\frac{c}{\nu}\right)^{-5} . \frac{c}{\nu^2} d\nu}{e^{h\nu/kT} - 1}$$

$$= \frac{8\pi hc . \nu^5/c^5 \cdot \frac{c}{\nu^2} d\nu}{e^{h\nu/kT} - 1} = \frac{8\pi h\nu^3 d\nu}{c^3 \left(e^{h\nu/kT} - 1\right)} \quad ----(5)$$

This represents plank's radiation law in terms of frequency ν.

Case I: (For small wavelengths)

Plank's formula reduces to Wien's formula for small wavelengths. When λ is small, $e^{hc/\lambda kT}$ is large as compared to 1.

∴ eq[n] (4) reduces to,

$$E_\lambda d\lambda = \frac{8\pi hc\lambda^{-5} d\lambda}{\left(e^{hc/\lambda kT}\right)} = \frac{8\pi hc}{\lambda^5} . e^{-hc/\lambda kT} d\lambda \quad ----(6)$$

This is Wien's law.

Case II: (For large wavelengths)

Plank’s formula reduces to Rayleigh Jean’s formula for larger wavelengths.

When λ is small, $e^{-hc/\lambda kT} = 1 + \frac{hc}{\lambda kT}$

∴ eq^n (4) reduces to,

$$E_\lambda d\lambda = \frac{8\pi hc\lambda^{-5} d\lambda}{1 + \frac{hc}{\lambda kT} - 1} = \frac{8\pi hc}{\lambda^5} . d\lambda . \frac{\lambda kT}{hc}$$

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} . d\lambda \;\; - - - - - - - - (7)$$

This is Rayleigh-Jean’s formula.

4. Linear momentum of photon in terms of wave vector:

The magnitude of linear momentum of a photon of wavelength λ is given by,

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \frac{h}{2\pi} \times k - - - - (1)$$

Where $2\pi/\lambda = k$ is magnitude of the wave vector

We know that, energy of photon is,

$$E = h\nu = \frac{hc}{\lambda}$$

Where frequency of radiation $\nu = c/\lambda$.

$$E = \frac{h}{2\pi} \times 2\pi\nu$$

$$E = \frac{h}{2\pi} \times w \;\; - - - - - - - (2)$$

Where $w = 2\pi\nu$ be angular frequency of particle.

As $\frac{h}{2\pi} = \hbar$

Then above eq^n (2) becomes,

$$E = \hbar w$$

5. Photoelectric Effect: -

Whenever light or electromagnetic radiations incident on a metal surface, it emits electrons. This process of emission of electrons from a metal surface when a light radiation of suitable frequency is incident, it is called as photoelectric effect. The surface which emits electrons, when illuminated with appropriate radiation is known as a photosensitive surface. The electrons emitted are known as photoelectrons. In case of alkali metals, photoelectric emission occurs under the action of visible light. Zinc, cadmium, magnesium etc., are sensitive to only ultraviolet light.

Experimental Set-up of Photoelectric Effect:

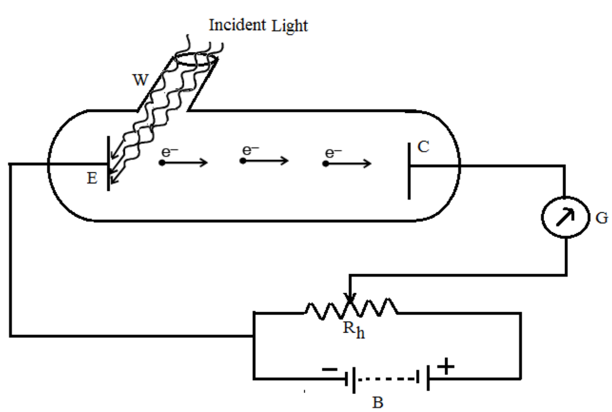

Fig (a)

The experimental arrangement to study photoelectric effect is as shown in Fig. (a). It consists of an evacuated glass tube with quartz window W containing a photosensitive metal plate called as emitter plate E and another metal plate called as collector plate C. Collector plate is connected to positive terminal and emitter plate E is connected to negative terminal of battery. When light is incident on plate E, the photoelectrons are ejected on plate E. These photoelectrons are attracted by positively charged collector plate C. Hence photoelectron flows in a direction from E to C in tube and photoelectric current flows from C to E. This current can be measured from the deflection of galvanometer G. It is found that the strength of photoelectric current increases as potential of collector plate is more and more positive with respect to emitter plate E. The result obtained can be summarised into four statements which are known as laws of photoelectric emission.

Laws of photoelectric Emission (Characteristics of Photoelectric effect):

i) For every metal, there is a particular minimum frequency of incident light below which there is no photoelectric emission, whatever be the intensity of radiations. This minimum frequency which can cause photoelectric emission is called threshold frequency. When frequency of incident light is greater than threshold frequency photoelectrons are emitted while below which photoelectrons are not emitted.

ii) The strength of photoelectric current is directly proportional to intensity of incident light, provided that the frequency of light is greater than the threshold frequency.

iii) The velocity and hence kinetic energy of emitted photoelectrons depends on frequency of incident light and nature of metal, independent of intensity of light.

iv) Photoelectric emission is an instantaneous process. The time lag between incidence of radiation and emission of electrons is never more than 3×10^{-9} sec.

Def^{n} of Threshold Frequency: The minimum value of frequency of incident radiation from which photoelectrons are emitted is called as threshold frequency. It is denoted by ν_o.

We know that, velocity = frequency × wavelength i.e. $c = \nu\lambda$

$\therefore$ Threshold frequency is $\nu_0 = \frac{c}{\lambda_0}$ where λ_0 is threshold wavelength.

Einstein's Photoelectric Equation: -

On the basis of plank's quantum theory, Einstein explained photoelectric effect in 1905. According to quantum theory, radiation is considered as a shower of particles called as photons. A photon of frequency ν carries an energy $h\nu$ and it travels with the velocity of light. Einstein assumed that when radiation is incident on a substance, there will be collision between the photons and the electrons in the substance. During the collision, the electron completely absorbs the energy of photon. The energy absorbed by electron can be used for two purposes, some part of energy can be used to escape the electron from the metal surface and remaining part of energy can be used to give the maximum kinetic energy to the electrons. Thus, the energy of the photon is equal to sum of maximum kinetic energy of ejected photoelectron and energy required to release the electron from metal surface.

Let ϕ be minimum amount of energy required to free the electron from metal surface called as photoelectric work function.

V_{max} is the maximum velocity of the electron.

$\therefore$ Photon energy = Photoelectric work function + kinetic energy

$$h\nu = \phi + \frac{1}{2}mV^2{}_{max} \quad ------(1)$$

This equation is called as Einstein's photoelectric equation.

Einstein's equation helps to explain all the characteristics of the photoelectric effect.

i) If the frequency of incident radiation is decreased, the kinetic energy of the photoelectrons will go on decreasing until finally it becomes zero for a certain frequency ν_0 i.e. threshold frequency. Thus K.E. = 0 when $\nu = \nu_0$

Substituting in eq^n (1) we get

$$h\nu_0 = \phi + 0$$

$$\phi = h\nu_0$$

Substituting this value in eq^n (1) we get,

$$h\nu = h\nu_0 + \frac{1}{2}mV^2{}_{max}$$

$$\frac{1}{2}mV^2{}_{max} = h(\nu - \nu_0) ------(2)$$

Let λ be wavelength of radiation and λ_0 be threshold wavelength.

We know that, $\nu = \frac{c}{\lambda}$ and $\nu_0 = \frac{c}{\lambda_0}$

Substituting this value in eq^n (2) we get,

$$\frac{1}{2}mV^2{}_{max} = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) ----(3)$$

From eq^n (2) it is clear that if $\nu > \nu_0$, Kinetic energy is positive and photoelectrons can be emitted, but $\nu < \nu_0$, Kinetic energy is negative and photoelectrons cannot be emitted i.e. when frequency of incident radiation is greater than the threshold frequency photoelectrons are emitted while below which photoelectrons are not emitted.

ii) According to the quantum theory, a more intense beam contains a greater number of photons. Consequently, there are a greater number of collisions between photons and electrons, liberating more photoelectrons. Therefore, an increase in the intensity of radiation increases the rate of photoelectric emission, which results into an increase in the photoelectric current.

iii) As ϕ is constant, the maximum kinetic energy of the photoelectrons increases with the frequency of incident radiation. Maximum kinetic energy does not depend upon the intensity of incident radiation.

iv) Emission of photoelectrons is a result of the collisions between photons and electrons. As soon as the radiation is incident on the emitting surface, such collisions begin to occur and photoelectrons are emitted. Thus, photoelectric effect is an instantaneous process.

Remarks: Def^n of photoelectric work function: The minimum amount of energy required to release the electron from metal surface is called as photoelectric work function.

6. **Compton Effect: -**

Statement: When the X-rays of a sharply defined frequency were incident on a material of low atomic number like carbon, they suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exists a line of longer wavelength. The change of wavelength is due to loss of energy of incident X-rays. This elastic interaction is known as Compton effect.

In the case of incoherent scattering, a scattered beam undergoes not only deviation in its direction but also change of wavelength occurs. In Compton effect, there is a change in wavelength of the scattered beam along with the change in its direction. Hence Compton effect is an incoherent scattering.

This effect was explained by Compton on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between X-ray photon and a loosely bound electron of the scatterer. In this process both momentum and energy are conserved. In the photon electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X-ray proceeds with less than the original energy.

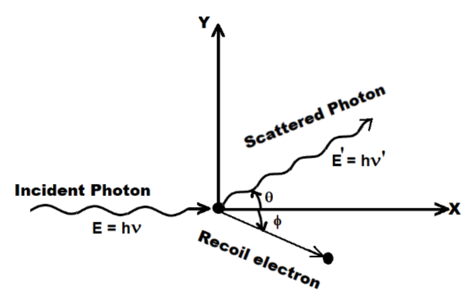

Fig. (a)

Suppose incident photon with energy hν and momentum $h/\lambda = h\nu/c$ strikes an electron at rest. The initial momentum of the electron is zero and its energy is only the rest mass energy m_0c^2. The scattered photon of energy hν′ and momentum hν′/c moves off in a direction inclined at an angle θ to the original direction. The direction acquires a momentum mv and moves at an angle ϕ to the original direction. The energy of the recoil electron is mc^2 as shown in fig (a).

According to the principle of conservation of energy,

$$h\nu + m_0c^2 = h\nu' + mc^2 \;-----(1)$$

Considering the x and y components of the momentum and applying the principle of conservation of momentum.

$$\frac{h\nu}{c} = \frac{h\nu'}{c}cos\theta + m\mathrm{v}cos\phi -----(2)$$

and

$$0 = \frac{h\nu'}{c}sin\theta - m\mathrm{v}sin\phi -----(3)$$

From eq^n(2)

$$h\nu = h\nu' cos\theta + m\mathrm{v}cos\phi$$

$$m\mathrm{v}c.\,cos\phi = h(\nu - \nu' cos\theta) -----(4)$$

From eqn(3)

$$m\mathrm{v}c.\,sin\phi = h\nu' sin\theta -----(5)$$

Squaring and adding eq^n(4) and eq^n(5)

$$m^2\mathrm{v}^2c^2cos^2\phi + m^2\mathrm{v}^2c^2sin^2\phi = h^2(\nu - \nu' cos\theta)^2 + h^2\nu'^2sin^2\theta$$

$$m^2\mathrm{v}^2c^2 = h^2\left(\nu^2 - 2\nu\nu' cos\theta + {\nu'}^2cos^2\theta\right) + h^2\nu'^2sin^2\theta$$

$$= h^2\nu^2 - 2h^2\nu\nu' cos\theta + h^2{\nu'}^2 = h^2\left(\nu^2 - 2\nu\nu' cos\theta + {\nu'}^2\right) ---(6)$$

From eq^n(1)

$$mc^2 = h(\nu - \nu') + m_0c^2$$

Squaring on both side

$$m^2c^4 = h^2(\nu - \nu')^2 + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$= h^2\left(\nu^2 - 2\nu\nu' + \nu'^2\right) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 - - - -(7)$$

Subtracting eq^n(6) and from eq^n(7)

$$m^2c^4 - m^2\mathrm{v}^2c^2 = -2\nu\nu'h^2 + 2\nu\nu'h^2cos\theta + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$m^2c^2(c^2 - \mathrm{v}^2) = -2\nu\nu'h^2(1 - cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 - - - (8)$$

By using mass relativistic formula,

$$m = \frac{m_0}{\sqrt{1 - \mathrm{v}^2/_{c^2}}}$$

Squaring on both sides

$$m^2 = \frac{m_0^2}{1 - \mathrm{v}^2/_{c^2}}$$

$$m^2(c^2 - \mathrm{v}^2) = m_0^2c^2$$

Multiply both side by c^2

$$m^2c^2(c^2 - \mathrm{v}^2) = m_0^2c^4$$

eq^n(8) becomes

$$m_0^2c^4 = -2\nu\nu'h^2(1 - cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$2\nu\nu'h^2(1 - cos\theta) = 2h(\nu - \nu')m_0c^2$$

$$\frac{h^2(1-cos\theta)}{h}=\frac{(\nu-\nu')m_0c^2}{\nu\nu'}$$

$$\frac{h(1-cos\theta)}{m_0c^2}=\frac{1}{\nu'}-\frac{1}{\nu}$$

$$\frac{c}{\nu'}-\frac{c}{\nu}=\frac{h(1-cos\theta)}{m_0c}$$

But λ=c/ν and λ'=c/ν'

$$\lambda'-\lambda=\frac{h}{m_0c}(1-cos\theta)$$

Therefore, the change in wavelength is

$$d\lambda=\frac{h}{m_0c}(1-cos\theta)----(9)$$

This is Compton effect. This relation shows that dλ is independent on wavelength of incident radiation as well as nature of scattering substance. It depends upon the angle of scattering only.

Case I: When θ=0, cosθ=1 then dλ=0.

Case II: When θ=90^0, cosθ=0 then $d\lambda=\frac{h}{m_0c}$

$$d\lambda=\frac{6.63\times10^{-34}}{9.11\times10^{-31}\times3\times10^8}=0.0243\text{Å}$$

This is known as Compton wavelength.

Case III: When θ=180^0, cosθ=-1 then $d\lambda=\frac{2h}{m_0c}=2\times0.0243=0.0486\text{Å}$

i.e. dλ is maximum at θ=180^0.

Numerical

Q.1 Calculate the frequency and energy of a photon of light of wavelength 6000Å. (Velocity of light is 3×10^8 m/s)

Sol^n: Given

λ =6000Å = 6000×10^{-10} m = 6×10^{-7}m

C = 3×10^8 m/s, h = 6.63×10^{-34} J-s , ν = ? and E = ?

We know that, C = $\nu\lambda$

$\therefore$ frequency of light is,

$$\nu = \frac{C}{\lambda} = \frac{3\times10^8}{6\times10^{-7}} = 0.5\times10^{15} = 5\times10^{14} Hz$$

Energy of photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{6\times10^{-7}}$$

$$E = \frac{19.89\times10^{-26}}{6\times10^{-7}} = 3.315\times10^{-19} J$$

2. Photo electric work function for a surface is 2.4eV, light of wavelength 6.8×10^{-7} m shines on the surface. Find threshold frequency.

Sol^n: Given: ϕ = 2.4eV = $2.4\times1.6\times10^{-19}$ J =3.84×10^{-19} J

λ = 6.8×10^{-7} m ν_o =?

Photoelectric work function is

$$\emptyset = h\nu_0$$

$$\nu_0 = \frac{\phi}{h} = \frac{3.84\times10^{-19}}{6.63\times10^{-34}}$$

$$\nu_o = 0.5792\times10^{15} = 5.792\times10^{14}\ Hz$$

3. The threshold wavelength for a certain metal is 3800 Å. Calculate the maximum kinetic energy of the photo electrons emitted when ultraviolet light of wavelength 2600 Å falls on it.

Sol^n: Given: λo = 3800 Å = 3.8×10^{-7} m , λ = 2600 Å = 2.6×10^{-7} m

KE $_{max}$ =?

We know that, the maximum kinetic energy of photoelectron is,

$$KE_{max} = h\,(\nu - \nu_o) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= 6.63\times10^{-34}\times3\times10^{8}\left(\frac{1}{3.8\times10-7} - \frac{1}{2.6\ \times10-7}\right)$$

$$= 19.89\times10^{-26}(0.3846\times10^{7} - 0.2631\times10^{7})$$

$$= 19.89\times10^{-26}(0.1215\times10^{7}) = 2.417\times10^{-19}J$$

4. A photon of energy 12 eV falls on metal surface whose work function is 4.15eV. Find kinetic energy.

Sol^n: Given: E = hν= 12 eV and ϕ= 4.15 eV, KE =?

Kinetic energy of photoelectron is

$$KE = h\nu - \phi$$

$$= 12 - 4.15 = 7.85\ eV$$

$$KE = 7.85\times1.6\times10^{-19}J = 12.56\ \times10^{-19}J$$

5. The surface temperature of the hot body is 1227^{o}C. Find the wavelength at which it radiates maximum energy. (Wien's constant b =2.898×10^{-3} m^{o}K)

Sol^n: Given

T = 1227^{o}C = 1227+273 = 1500 ^{o}K, λ = ?

According to Wien's displacement law

$$\lambda_m\,T = b$$

$$\lambda_m = \frac{b}{T} = \frac{2.898\times10^{-3}}{1500}$$

$$\lambda_m = 0.001932\times10^{-3} = 1.932\times10^{-6}m$$

Multiple Choice Question

1. A perfectly black body is one which--------

(a) absorbs all the radiant heat incident on it

(b) reflect all the radiant heat incident on it

(c) transmit all the radiant heat incident on it

(d) All of these

2. Energy of photon is -------

(a) $E = h\nu$ (b) $E = hc/\lambda$ (c) $E = Pc$ **(d) All of these**

3. According to Wien's displacement law, wavelength of radiation is ---

(a) directly proportional to absolute temperature of body

(b) inversely proportional to absolute temperature of body

(c) directly proportional to square of absolute temperature of body

(d) inversely proportional to square of absolute temperature of body

4. In the spectrum of energy distribution of black body radiation, area under each curve is-----

(a) directly proportional to absolute temperature of body

(b) inversely proportional to absolute temperature of body

(c) directly proportional to fourth power of absolute temperature of body

(d) inversely proportional to fourth power of absolute temperature of body.

5. According to Plank's quantum theory, light radiation consists of group of small energy packets called as ----

(a) fermions (b) **quanta** (c) wave packets (d) bosons.

6. According to plank's radiation law, energy radiated per unit volume by a cavity of a blackbody is ------

(a) inversely proportional to wavelength of radiation.

(b) inversely proportional to square of wavelength of radiation.

(c) inversely proportional to fourth power of wavelength of radiation.

(d) inversely proportional to fifth power of wavelength of radiation.

7. For smaller wavelength, Plank's radiation law reduces to --------

(a) Wien's law (b) Rayleigh Jean's law

(c) Kirchhoff's law (d) Photoelectric law

8. For larger wavelength, Plank's radiation law reduces to --------

(a) Wien's law **(b) Rayleigh Jean's law**

(c) Kirchhoff's law (d) Photoelectric law

9. The value of Wien's constant is --------.

(a) 2.898×10^{-3} mk (b) $2.898\times10^{+3}$ mk

(c) 2.898×10^{-6} mk (d) $2.898\times10^{+6}$ mk

10. If the wavelength corresponding to maximum energy radiated from the moon is 14 micron and Wien's constant is 2.8×10^{-3} mk, then temperature of the moon is ------

(a) 100°K **(b) 200°K** (c) 2000°K (d) 400°K

11. The process in which photoelectrons are emitted due to electromagnetic radiation is called as -----

(a) primary emission (b) secondary emission

(c) thermionic emission **(d) photoelectric emission**

12. In photoelectric effect, photoelectrons are emitted from metal surface,

(a) only if the frequency of radiation is below the threshold frequency

(b) only if the frequency of radiation is above the threshold frequency

(c) only if the temperature of surface is low

(d) at a rate that is independent of the nature of metal.

13. The minimum energy required to remove an electron from metal surface is called

(a) Stopping potential (b) Kinetic energy

(c) Work function (d) None of these

14. The strength of photoelectric current depends upon-----

(a) frequency of incident radiation **(b) intensity of incident radiation**

(c) Both frequency and intensity of radiation (d) none of these

15. The kinetic energy of emitted photoelectrons depends only on ----

(a) Frequency of incident radiation (b) intensity of incident radiation

(c) Nature of metal **(d) both (a) and (c)**

16. Einstein's photoelectric equation is,

(a) $h\nu = \phi - \frac{1}{2}mV^2{}_{max}$ (b) $h\nu = \frac{1}{2}mV^2{}_{max}$

(c) $h\nu = \phi + \frac{1}{2}mV^2{}_{max}$ (d) $2h\nu = \phi + mV^2{}_{max}$

17. When light of 2.5eV falls on a metal surface, maximum kinetic energy of electron is 1.5eV then work function of metal is -----

(a) 1eV (b) 2eV (c) 0.5eV (d) 1.5eV

18. The radiation is incident on the metallic surface, if the work function is 1.32×10^{-12} J. The threshold frequency is ------

(a) 2×10^{21}Hz (b) 2×10^{-21}Hz (c) 20×10^{21}Hz (d) 20×10^{-21}Hz

19. Photoelectric effect was explained by ------

(a)Hertz (b) Faraday (c) Plank **(d) Einstein**

20. The phenomenon of photoelectric effect is -----

(a) adiabatic process **(b) instantaneous process**

(c) isothermal process (d) spontaneous process

21. The Compton effect can be explained on the basis of---------

a) Wave nature of light **b) Quantum theory of light**

c) Ray optics d) Wave optics

22. What kind of photon is required for the Compton effect to occur?

a) Visible Light Photon **b) X-ray Photon**

c) Infrared d) UV Photon

23. The expression for Compton shift is ------------

a) $\frac{h}{m_0C}\ cos\theta$ b) $\frac{h}{m_0C}\ sin\theta$

c) $\boldsymbol{\frac{h}{m_0 C}(1 - cos\theta)}$ d) $\frac{h}{m_0 C}(1 - sin\theta)$

24. In Compton effect, on scattering which of the following is conserved?

a) Energy b) momentum c) Wavelength **d) Both a and b**

25. Compton wavelength depends on -------

a) Incidence radiation b) Nature of scattering substance

c) angle of scattering d) amplitude of frequency